Worksheet 6

Name: ______________________ Score: __________

1. Let W be the subspace spanned by the vectors. Find a basis for the orthogonal complement $W^\perp$ of W.

(a)

$$\begin{bmatrix} -5 \\ -25 \\ -1 \\ 24 \end{bmatrix}, \quad \begin{bmatrix} 4 \\ 20 \\ 1 \\ -19 \end{bmatrix}.$$

Solution:

Let A be the matrix with columns the above vectors:

$$\begin{bmatrix} -5 & 4 \\ -25 & 20 \\ -1 & 1 \\ 24 & -19 \end{bmatrix}$$

Reduced row echelon form (RREF) of A^T:

$$\begin{bmatrix} 1 & 5 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Basis for $W^\perp = (\mathrm{Col}A)^\perp = \mathrm{Nul}A^T$:

$$\begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 5 \\ 0 \\ -1 \\ 1 \end{bmatrix}.$$

(b)

$$\begin{bmatrix} -3 \\ -15 \\ -15 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 10 \\ 10 \\ 1 \end{bmatrix}.$$

Solution:

Let A be the matrix with columns the above vectors:

$$\begin{bmatrix} -3 & 2 \\ -15 & 10 \\ -15 & 10 \\ 0 & 1 \end{bmatrix}$$

Reduced row echelon form (RREF) of A^T:

$$\begin{bmatrix} 1 & 5 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Basis for $W^\perp = (\text{Col}A)^\perp = \text{Nul}A^T$:

$$\begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \qquad \begin{bmatrix} -5 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

(c)

$$\begin{bmatrix} 0 \\ 4 \\ 0 \\ 16 \end{bmatrix}, \qquad \begin{bmatrix} 4 \\ -5 \\ 3 \\ 2 \end{bmatrix}, \qquad \begin{bmatrix} -2 \\ -2 \\ 2 \\ -12 \end{bmatrix}.$$

Solution:
Let A be the matrix with columns the above vectors:

$$\begin{bmatrix} 0 & 4 & -2 \\ 4 & -5 & -2 \\ 0 & 3 & 2 \\ 16 & 2 & -12 \end{bmatrix}$$

Reduced row echelon form (RREF) of A^T:

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Basis for $W^\perp = (\text{Col}A)^\perp = \text{Nul}A^T$:

$$\begin{bmatrix} -4 \\ -4 \\ -2 \\ 1 \end{bmatrix}.$$

2. Decide whether the linear system of equations $A\vec{x} = \vec{b}$ has a solution. If not, find a least squares solution.

(a)

$$A = \begin{bmatrix} -3 & 1 \\ 0 & 1 \\ 0 & 2 \end{bmatrix}, \qquad \vec{b} = \begin{bmatrix} -2 \\ -2 \\ 3 \end{bmatrix}$$

Solution: The system $A\vec{x} = \vec{b}$ does not have solutions. The normal equations are:

$$A^T A = \begin{bmatrix} 9 & -3 \\ -3 & 6 \end{bmatrix}$$

$$A^T \vec{b} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

The reduced row echelon form (RREF) of $A^T A\vec{x} = A^T\vec{b}$ is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 2 & 1 \\ -4 & -3 \\ 2 & 1 \end{bmatrix}, \qquad \vec{b} = \begin{bmatrix} 2 \\ -4 \\ -2 \end{bmatrix}$$

Solution: The system $A\vec{x} = \vec{b}$ does not have solutions. The normal equations are:

$$A^T A = \begin{bmatrix} 24 & 16 \\ 16 & 11 \end{bmatrix}$$

$$A^T \vec{b} = \begin{bmatrix} 16 \\ 12 \end{bmatrix}$$

The reduced row echelon form (RREF) of $A^T A\vec{x} = A^T\vec{b}$ is

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \end{bmatrix}$$

(c)

$$A = \begin{bmatrix} 2 & 1 \\ -4 & -5 \\ -1 & -1 \\ -2 & 2 \end{bmatrix}, \qquad \vec{b} = \begin{bmatrix} 5 \\ 3 \\ 3 \\ -4 \end{bmatrix}$$

Solution: The system $A\vec{x} = \vec{b}$ does not have solutions. The normal equations are:

$$A^T A = \begin{bmatrix} 25 & 19 \\ 19 & 31 \end{bmatrix}$$

$$A^T \vec{b} = \begin{bmatrix} 3 \\ -21 \end{bmatrix}$$

The reduced row echelon form (RREF) of $A^T A\vec{x} = A^T\vec{b}$ is

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$